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FRICTION FACTOR MEASUREMENTS IN A RECTANGULAR CHANNEL WITH WALLS OF IDENTICAL AND NON-IDENTICAL ROUGHNESS

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Abstract—Friction factor measurements have been made in a rectangular channel formed essentially by two flat plates roughened by transverse isolated square ribs. When the plates are not identically roughened a theory allows the friction factor of each surface to be separated and applied to other passages having ribs with the same ratios of pitch to height and height to equivalent diameter. The experiments indicate that the application of this theory produces friction factors which, compared with those for identically roughened plates for which no transformation is required, vary similarly with Reynolds number but differ in magnitude, but by less than 10 per cent over the range covered. The results show only moderate agreement with similar results obtained from an annular channel.

NOMENCLATURE

- a, flow area;
- d, equivalent diameter = 4 × flow area/wetted perimeter;
  - d_1, d_2 equivalent diameters for surfaces 1 and 2, respectively, in channel containing either or both surfaces;
- e, height of rib;
- f, friction factor;
  - f_1, f_2 friction factors for channels fully roughened with surfaces 1 and 2 respectively;
  - $f_{1(2)}$ friction factor of surface 1 in presence of surface 2;
  - f_{12} untransformed friction factor when mixed surfaces are present;
- h, heat-transfer coefficient = $q/\Delta\theta$;
- l, plate width;
- p, pitch of ribs;
- P, static pressure;
- q, heat flux;
- Re, Reynolds number = $\rho\bar{u}d/\mu$;
- St, Stanton number = $h/\rho\bar{u}\sigma$;
- $\bar{u}$, mean velocity in the x-direction;
- u, v, w, point velocities in the x, y, z directions respectively (see Appendix 1);
- W, mass flow rate;
- y_m, position of surface of maximum velocity;
- dP/dx, static pressure gradient along channel;
- $\Delta\theta$, temperature difference between plate and bulk mean air temperature;
- μ, viscosity;
- ρ, density of air;
- σ, specific heat at constant pressure;
- τ, surface shear (force/unit area).

INTRODUCTION

A THEORY by Hall [1] allows measurements of forced convective heat transfer and friction factor for longitudinal flow in a parallel passage to be applied to other passages with different shapes and/or boundary conditions; the term "boundary conditions" refers to the imposed distribution of surface heat flux as well as the proportions of smooth and roughened surfaces.

The theory is therefore very useful in allowing experiments to be carried out on a convenient arrangement, such as an annulus with the inner surface roughened and the outer surface smooth, rather than on one which may be of greater practical interest, such as, for instance, an internally roughened pipe or a cluster of artificially roughened rods in a smooth channel.

The basic assumption of the transformation theory is that the surface of zero shear, which for want of other information is assumed to coincide with the surface of zero velocity gradient, is a boundary within which the structure of the turbulence is unaffected by changes in the structure occurring beyond the surface. Thus in a channel formed by two infinite flat plates, one rough and one smooth (see Fig. 1a) the flow between the region of the rough surface and the plane of zero velocity gradient would be unaffected if the smooth plate were removed and replaced by an identical rough plate at the same distance as the first rough plate is from the plane of zero velocity gradient (see Fig. 1b).

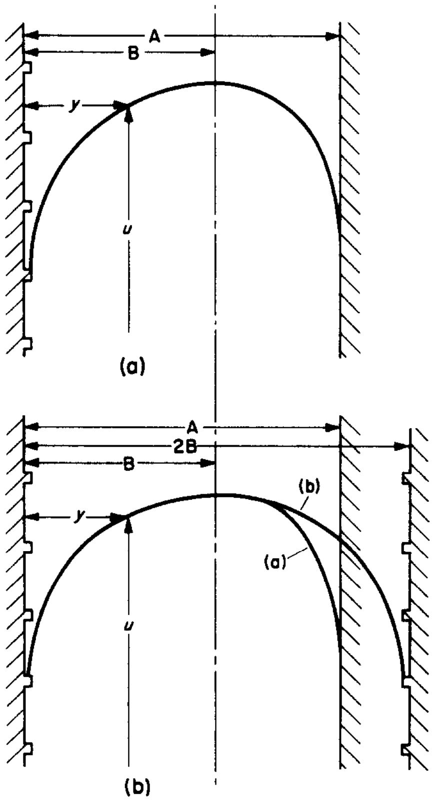

FIG. 1. Pictorial presentation of the transformation theory.

To test this theory a rectangular channel formed by two plates whose roughness can be independently varied is convenient, for not only should the results obtained from one surface be independent of the form of the other if the theory is correct, but should be the same as those obtained when both plates are identically roughened, a configuration for which no transformation need be performed. This in fact is the system adopted for the present investigation but comparison is also made with the results from other work [2] carried out in this laboratory on a markedly different arrangement, namely an annulus with smooth outer wall and rough inner wall.

For reasons given later, emphasis has been put on the study of friction factor. In this work, except in the particular case where a comparison is made with the results of other workers of the friction factor for a completely smooth channel, the analysis has been confined to the region in which the velocity gradient normal to the smooth side walls is zero since by using an integrated mass flow rate over this region the influence of the side walls can be eliminated.

The roughness investigated is in the form of isolated transverse square-section ribs (Fig. 2) whose ratio of pitch to height has been kept constant at 10 but whose size relative to the equivalent diameter* has been varied from 0·5 to 1·5 per cent.

APPARATUS

The rectangular channel was 5-ft long in the direction of the flow, 6-in wide and 0·54 in between the plates as measured from the rib tips. Air at approximately atmospheric pressure and

* For two infinite flat plates to which the present measurements apply, the equivalent diameter is four times the distance between the base of the roughness on one plate and the surface of no shear.

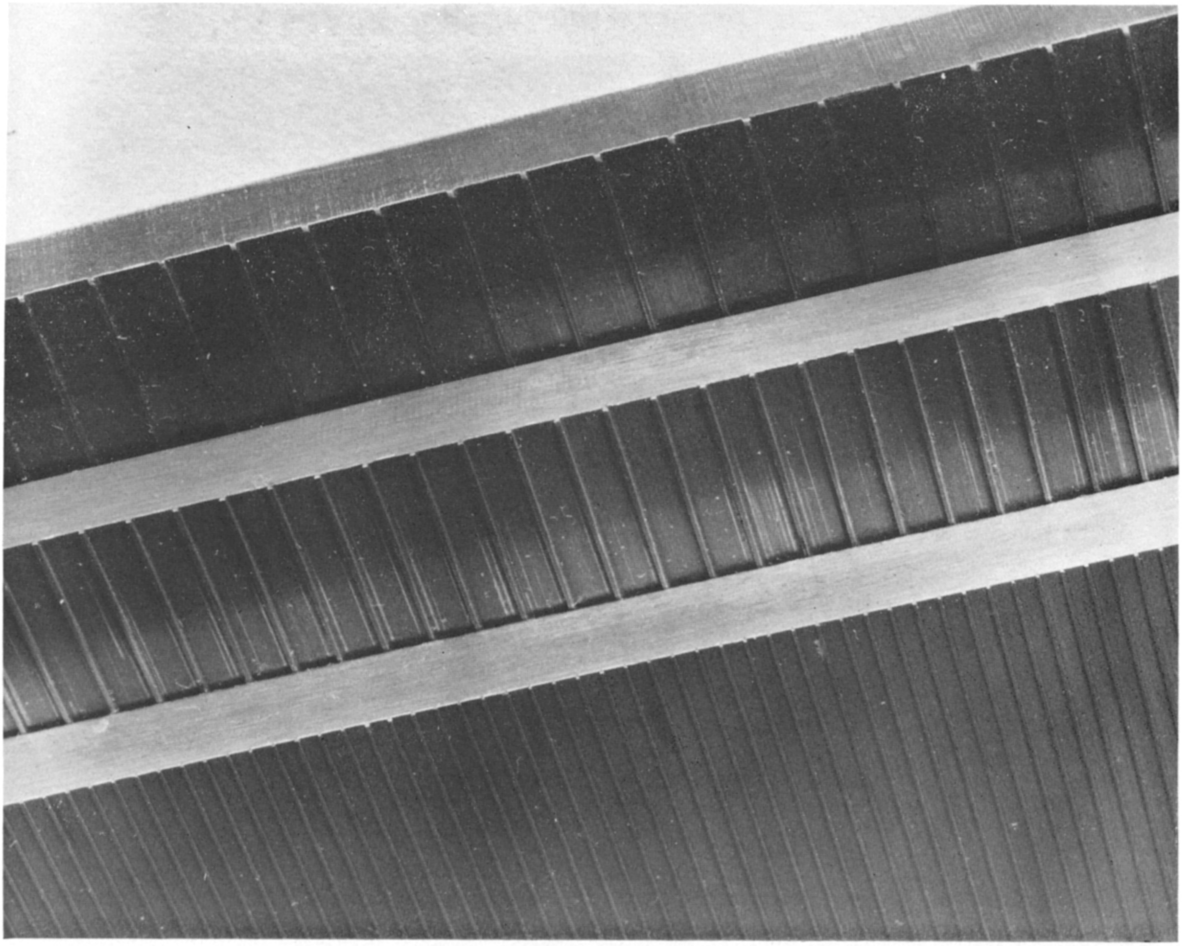

FIG. 2. Section of three roughened plates.

temperature was applied by a blower capable of delivering 0·6 lb/s.

The pressure drop along the channel was measured with pressure tappings located at 2-in intervals along one of the side walls. Velocity and temperature distributions were measured at a plane 44 in downstream from the inlet by a pair of Pitot tubes containing thermocouples entering from a side wall. The tubes, 0·056 in o.d., were separated at their mouths by 0·200 in between centres but their stems were held rigidly together on a common plate of streamlined shape. The separation of the mouths allowed measurements to be made close to each wall. The position of the Pitot tubes was measured in the 6-in direction by means of a scale divided into tenths of an inch and in the 0·54-in direction by a dial gauge divided into thousandths of an inch. The zero positions of the probes relative to the plates were determined by electrical contact. Measurements with the probes were made at 0·020-in spacing giving an overlap of five points in the centre of the channel.

The heat-transfer measurements were confined to the case of both plates smooth. The interesting cases of different heat fluxes and different roughnesses with equal or different heat fluxes on opposite plates could not be satisfactorily performed because of transfer of heat from the hotter to the colder plate via the thermally insulated side walls.

THE EXPERIMENTS

Four pairs of plates were made, one pair smooth, and the other three roughened with square ribs of mean height 0·0060, 0·0121 and 0·0184 in and with tolerances* on individual rib heights of $\pm 0{\cdot}0002$ $\pm 0{\cdot}0005$ and $\pm 0{\cdot}0016$ in respectively. Other dimensions were to within $\pm$ 0·001 in.

Friction factors and velocity profiles were measured over a range of Reynolds numbers from 2×10^4 to 2×10^5 for all combinations of roughness heights (including zero) taken two at a time.

Stanton numbers were determined for the smooth plates only. During these experiments the ratio of wall to bulk mean absolute temperature was kept approximately constant at 1·15 by adjusting the heat flux to counteract the controlled changes in flow rate.

CALCULATION OF RESULTS

For each asymmetric assembly the surface of maximum velocity, assumed to be the surface of no shear, was obtained from the velocity distribution (a typical one is shown in Fig. 3) and was

* For the 0·006-in and 0·012-in ribs the tolerances quoted are the maximum since every rib was inspected and those initially outside these tolerances were individually altered. For the 0·018-in ribs the value quoted is one standard deviation.

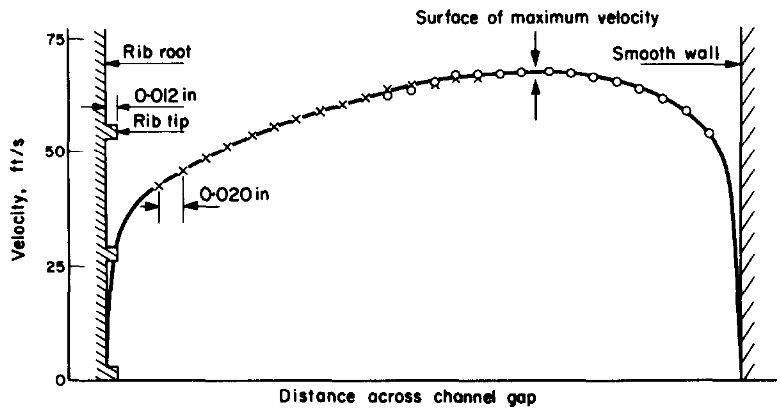

FIG. 3. Typical velocity profile between plates.

used to divide the channel into two flow areas. A friction factor, equivalent diameter and Reynolds number were calculated for each of the two rough surfaces, (see Appendix I) and the friction factor and equivalent diameter both plotted as functions of Reynolds number. Best lines were drawn through each set of points and values of $f_{1(2)}$ (the friction factor of surface 1 in the presence of surface 2 as obtained from the transformation) and d_1 picked off at particular Reynolds numbers.

Since the transformation is not needed for symmetrical assemblies the friction factors obtained from these tests may be used to compare with the values obtained from the transformation. This is done by plotting these friction factors (Table 3) as a function of e/d at particular Reynolds numbers. By interpolation values of f_1 at various e/d may be obtained.

The Stanton numbers were calculated from the formula: $St = qa/W\sigma\Delta\theta$ where a is the total flow area. The heat flux q was corrected for heat losses from the rig, the correction being obtained from a preliminary experiment. The mass flow rate W was obtained from orifice plate readings. In determining $\Delta\theta$, the air temperature at the measuring point was calculated from the temperature rise per foot (obtained from q and W) and the inlet air temperature. A mean plate temperature at the measurement point was obtained from a best straight line drawn through the experimental readings of the plate thermocouples in this region.

Table 1. Experimental values of friction factor and Reynolds number obtained from symmetrical assemblies

$e/d \times 1000$	Reynolds number $\times 10^{-5}$	Friction factor
0	0·502	0·00650
	0·866	0·00595
	1·202	0·00554
	1·712	0·00521
5·40	0·251	0·0147
	0·353	0·0157
	0·502	0·0171
	0·800	0·0179
	1·212	0·0181
10·67	0·236	0·0245
	0·343	0·0243
	0·520	0·0249
	0·728	0·0256
	1·228	0·0250
15·65	0·205	0·0258
	0·398	0·0284
	0·480	0·0271
	1·103	0·0276

RESULTS AND DISCUSSION

The Stanton number and its variation with Reynolds number for the two smooth plates (excluding the side walls which were unheated) is in very good agreement with the Colburn equation for a smooth circular pipe (Fig. 4).

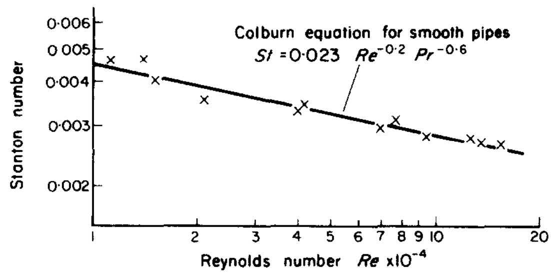

FIG. 4. Relation between Stanton number and Reynolds number for two smooth plates.

The friction factor and its variation with Reynolds number for the completely smooth channel, but including the side walls, is also in agreement with the results of other workers [3] for similar systems and for circular pipes (Fig. 5).

The friction factors for the roughened surfaces are presented in Fig. 6. Cross plotting gives Fig. 7. The curves in Fig. 6(b) have been derived from the results of all the experiments in which rough plates were present, including those with a pair of identically roughened plates, since if the transformation theory is correct all results for a given value of e/d will be identical apart from experimental error, whereas if the theory is incorrect no single combination of mixed surfaces can be considered to give representative transformed data. Therefore, in either case, it is

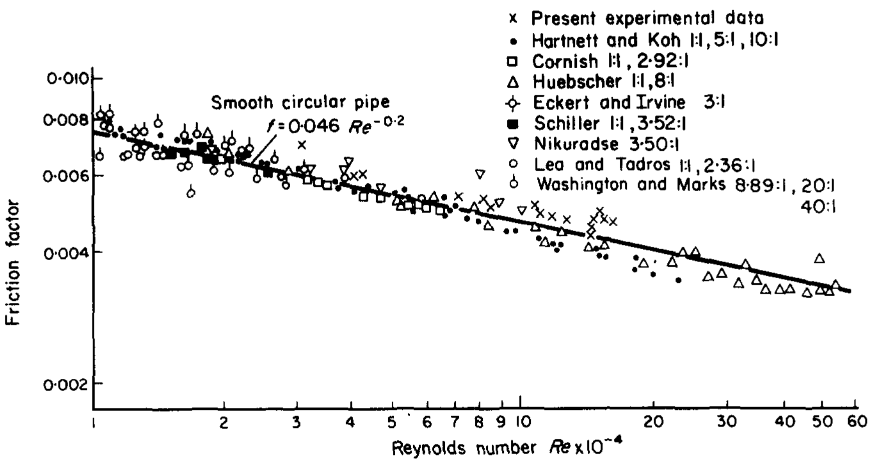

FIG. 5. Friction factor variation with Reynolds number comparison with all other available data for smooth rectangular channels.

appropriate to take mean values. For a given roughness height e, the value of e/d varies slightly with the roughness height of the facing plate. In presenting Fig. 6(b), results have been adjusted to common values of e/d. The equivalent diameter used in calculating the results is measured from the root of the ribs. There is no conclusive evidence for this choice but with these widely spaced ribs, where the flow reattaches to the surface behind each rib, it seems reasonable to continue to regard the base surface as the boundary of the flow rather than postulate a new effective bounding plane passing, for example, through the tips of the ribs.

The form of the results in Fig. 7 is very similar to that obtained from the experiments on the annular channel with roughened inner cylinder [2], namely that the friction factor is independent of Reynolds number for the large ribs and for the smaller ribs at the high Reynolds numbers, indicating that the height of the rib relative to the boundary layer whose thickness decreases with increasing Reynolds number, is the relevant parameter for describing a fully roughened surface.

The shape of the curves for transformed data obtained with non-identically roughened plates is the same as for data obtained with identically roughened plates and hence requiring no transformation. This is an important conclusion for it means that the variation with Reynolds number is not affected by applying the transformation whatever the theory may be doing to the level of friction factor.

The variation of friction factor with the ratio of rib height to equivalent diameter is shown in Fig. 6. The decrease in gradient with increasing rib height, exhibited by the data for the identically roughened plates as well as by the transformed data, is similar to the trend for the annular data at the same pitch-to-height ratio.

Having established that the effect of Reynolds number on friction factor is little affected by the transformation it is now important to consider how the level of friction factor is affected. This is shown by the plot in Fig. 8 of $f_{1(2)}/f_1$ against f_2/f_1, i.e. of the ratio of the transformed friction factor for roughness 1 (as defined by the ratios of pitch and height to equivalent diameter) in the presence of roughness 2 to the friction factor*

* Since e/d is a dependent variable in experiments with mixed surfaces, results for f_2 and f_1 must be obtained at the corresponding e/d by interpolation from the fixed e/d given in Fig. 7.

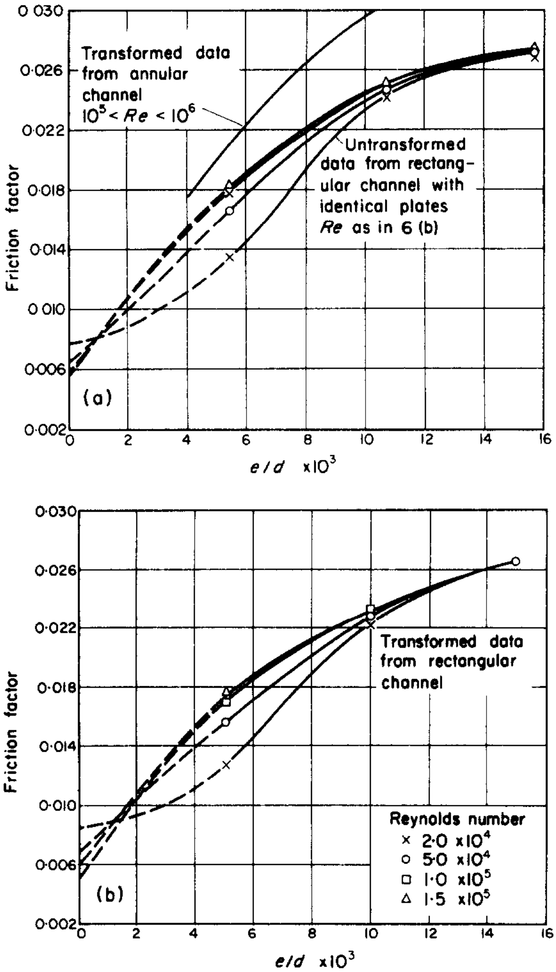

FIG. 6. Variation of friction factor with ratio of rib height to equivalent diameter for $p/e = 10$.

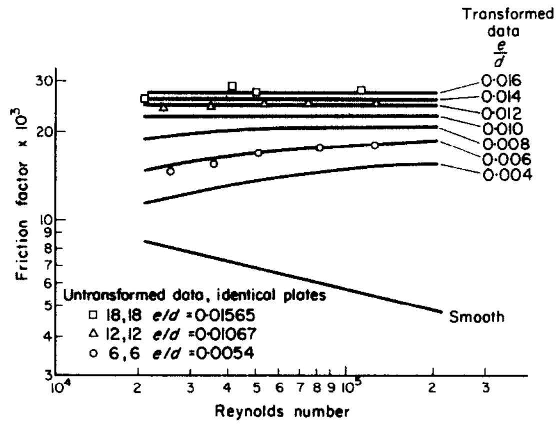

FIG. 7. Variation of friction with Reynolds number for $p/e = 10$.

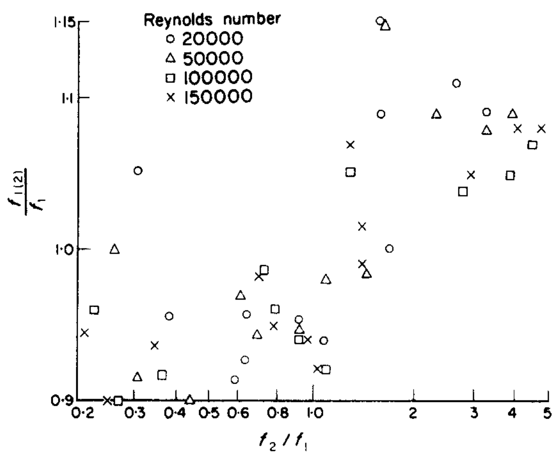

FIG. 8. Breakdown of the Hall transformation.

of the channel fully roughened with roughness 1, against the ratio of the friction factors for channels fully roughened with roughnesses 2 and 1 respectively. If the transformation theory is correct $f_{1(2)}$ should be the same as f_1 irrespective of the roughness of the opposite surface so that Fig. 8 should give a horizontal line at unit height. Significance tests (see Appendix 2) show that the transformation theory gives a significantly lower value when f_2 is less than f_1 (i.e. when surface 2 is smoother than surface 1) and a significantly higher value when f_2 is greater than f_1 (i.e. when surface 2 is rougher than surface 1). Therefore there seems to be little doubt that the theory is not a complete success. However, the magnitude of the failure is such that the theory gives values that for the particular configuration chosen (viz. rectangular channel and square ribs with pitch/height of 10) are within 10 per cent of the actual values.

The discrepancy between theory and experiment indicates breakdown of the assumption that the maximum velocity surface coincides with the surface of no shear, since the calculation

Table 2. Experimental values of friction factor and Reynolds number obtained from asymmetric assemblies, by means of transformation theory

Assembly	$e/d_1 \times 1000$	Re $\times 10^{-5}$	$f_{1(2)}$	$e/d_2 \times 1000$	$Re \times 10^{-5}$	$f_{2(1)}$	$Re \times 10^{-5}$	Untransformed friction factor f_{12}
0,6	0	0·222	0·0082	4·76	0·291	0·0124	0·257	0·0103
	0	0·269	0·0077	4·49	0·405	0·0132	0·337	0·0104
	0	0·371	0·0071	4·33	0·610	0·0140	0·490	0·0105
	0	0·540	0·0064	4·05	1·092	0·0151	0·816	0·0108
	0	0·832	0·0063	4·14	1·524	0·0146	1·178	0·0104
	0	1·069	0·0061	4·09	2·015	0·0147	1·542	0·0103
0,12	0	0·194	0·0091	8·45	0·292	0·0219	0·243	0·0152
	0	0·181	0·0089	8·30	0·316	0·0186	0·248	0·0137
	0	0·212	0·0085	7·92	0·426	0·0203	0·319	0·0144
	0	0·239	0·0079	8·03	0·450	0·0189	0·345	0·0133
	0	0·334	0·0075	7·81	0·701	0·0197	0·517	0·0136
	0	0·486	0·0070	7·69	1·076	0·0193	0·781	0·0131
	0	0·758	0·0069	7·74	1·648	0·0187	1·203	0·0128
0,18	0	0·305	0·0079	11·43	0·663	0·0255	0·493	0·0169
	0	0·384	0·0072	11·26	0·923	0·0244	0·675	0·0163
	0	0·575	0·0071	10·93	1·360	0·0239	0·967	0·0154
	0	0·736	0·0066	10·9	1·843	0·0237	1·300	0·0151
6,12	5·96	0·213	0·0170	9·63	0·258	0·0223	0·235	0·0197
	5·90	0·313	0·0176	9·70	0·370	0·0226	0·341	0·0201
	5·80	0·432	0·0186	9·80	0·503	0·0230	0·407	0·0208
	5·85	0·725	0·0193	9·77	0·868	0·0232	0·796	0·0213
	5·79	1·169	0·0195	9·89	1·351	0·0234	1·260	0·0214
6,18	6·41	0·203	0·0157	13·67	0·272	0·0245	0·238	0·0200
	6·26	0·387	0·0183	13·93	0·511	0·0256	0·449	0·0220
	6·36	0·453	0·0179	13·72	0·529	0·0259	0·514	0·0219
	6·29	0·703	0·0178	14·02	0·889	0·0272	0·796	0·0225
	6·07	1·016	0·0194	14·27	1·288	0·0254	1·152	0·0224
12,18	10·8	0·244	0·0237	15·6	0·250	0·0256	0·247	0·0246
	11·4	0·283	0·0221	14·9	0·325	0·0251	0·304	0·0237
	10·7	0·428	0·0255	15·5	0·452	0·0252	0·438	0·0254
	11·2	0·572	0·0237	15·2	0·631	0·0262	0·601	0·0250
	11·1	0·812	0·0238	15·3	0·874	0·0260	0·843	0·0249
	11·1	1·185	0·0237	15·3	1·288	0·0253	1·236	0·0245

is merely a force balance between the two flow zones (Appendix 1). If the difference between the positions of the two surfaces is the sole cause of the discrepancy, the position of the surface of no shear can be calculated from knowledge of the position of the maximum velocity surface and the ratio of the maximum velocity to the mean velocities on either side of it. These quantities are plotted in dimensionless form in Figs. 9 and 10, as functions of f_2/f_1. The choice of the latter variable was based on the idea that the drag on the surface would be the primary source of variation in the flow parameters and that the detailed roughness shape would be a second order effect only. The small scatter of the points about the curves supports this view. (The distances to the maximum velocity surface for $f_2/f_1 < 1$ are the complements of those for $f_2/f_1 > 1$ corresponding pairs having been taken from a single determination. Both values have been included in order to facilitate the drawing of the curve.)

Table 3. Comparison between friction factors from transformation and expected values

Reynolds number	Assembly	e/d for surface 1 × 1000	$f_{1(2)}$	Predicted f from symmetrical assembly, f_1	Predicted f for surface 2 f_2	$f_{1(2)}/f_1$	f_2/f_1
2×10^4	0,6	0	0·0085	0·0078	0·0123	1·09	1·58
		4·77	0·0114	0·0123	0·0078	0·927	0·634
	0,12	0	0·0087	0·0078	0·0206	1·11	2·64
		8·5	0·0197	0·0206	0·0078	0·956	0·379
	0,18	0	0·0086	0·0078	0·0255	1·09	3·27
		12·0	0·0268	0·0255	0·0078	1·05	0·306
	6,12	5·92	0·0167	0·0145	0·0227	1·15	1·57
		9·47	0·0217	0·0227	0·0145	0·956	0·638
	6,18	6·4	0·0156	0·0156	0·0263	1·00	1·69
		13·4	0·0240	0·0263	0·0156	0·914	0·593
	12,18	11·1	0·0233	0·0248	0·0270	0·939	1·09
		15·3	0·0257	0·0270	0·0248	0·952	0·918
5×10^4	0,6	0	0·0068	0·0065	0·0149	1·09	2·29
		4·44	0·0134	0·0149	0·0065	0·90	0·436
	0,12	0	0·0070	0·0065	0·0214	1·08	3·29
		8·01	0·0196	0·0214	0·0065	0·915	0·304
	0,18	0	0·0071	0·0065	0·0252	1·09	3·88
		11·4	0·0253	0·0252	0·0065	1·00	0·258
	6,12	5·92	0·0167	0·0145	0·0236	1·15	1·63
		9·75	0·0229	0·0236	0·0145	0·969	0·614
	6,18	6·27	0·0181	0·0184	0·0266	0·984	1·45
		13·8	0·0251	0·0266	0·0184	0·944	0·691
	12,18	11·1	0·0234	0·0249	0·0271	0·980	1·09
		15·2	0·0257	0·0271	0·0249	0·948	0·919
10^5	0,6	0	0·0060	0·0047	0·0157	1·04	2·75
		4·2	0·0144	0·0157	0·0057	0·917	0·363
	0,12	0	0·0060	0·0057	0·0216	1·05	3·79
		7·7	0·0194	0·0216	0·0057	0·898	0·264
	0,18	0	0·0062	0·0051	0·0253	1·07	4·44
		11·1	0·0243	0·0253	0·0057	0·960	0·225
	6,12	5·8	0·0197	0·0187	0·0240	1·052	1·28
		9·85	0·0231	0·0240	0·0187	0·961	0·779
	6,18	6·20	0·0192	0·0194	0·0271	0·991	1·40
		14·3	0·0267	0·0271	0·0194	0·986	0·715
	12,18	11·1	0·0234	0·0254	0·0274	0·921	1·08
		15·3	0·0258	0·0274	0·0254	0·941	0·926
$1·5 \times 10^5$	0,6	0	0·0056	0·0053	0·0156	1·05	2·94
		4·09	0·0146	0·0156	0·0053	0·936	0·340
	0,12	0	0·0057	0·0053	0·0214	1·08	4·04
		7·52	0·0193	0·0214	0·0053	0·901	0·248
	0,18	0	0·0057	0·0053	0·0253	1·08	4·77
		11·0	0·0239	0·0253	0·0053	0·944	0·210
	6,12	5·77	0·0200	0·0187	0·0243	1·07	1·30
		10·00	0·0231	0·0243	0·0187	0·950	0·769
	6,18	6·15	0·0196	0·0193	0·0271	1·015	1·405
		14·3	0·0267	0·0271	0·0193	0·984	0·711
	12,18	11·1	0·0234	0·0254	0·0274	0·921	1·038
		15·3	0·0258	0·0274	0·0254	0·942	0·964

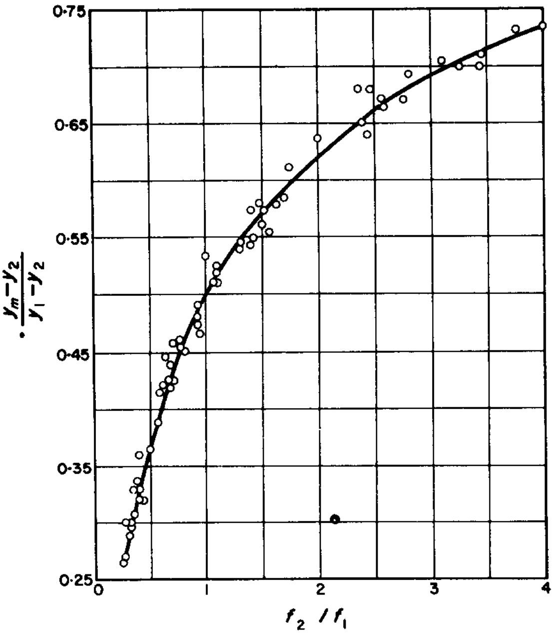

FIG. 9. Variation of the position of maximum velocity with ratio of friction factors of non-identical plates.

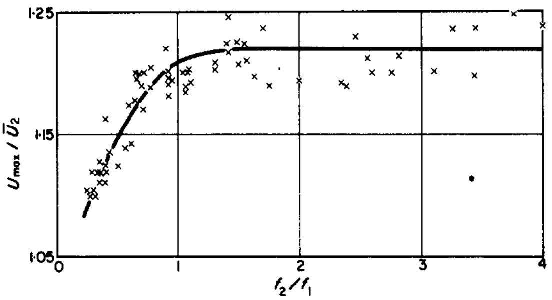

FIG. 10. Ratio of maximum to mean velocity in one zone as function of friction factor ratio.

The required shift in the position of no shear to make the Hall transformation produce correct answers is towards the smoother surface. The reduction in friction factor on the smoother side is greater than the increase on the rougher side since the maximum velocity surface is nearer the smoother side. For example when the smoother surface is smooth and $f_2/f_1 = 3{\cdot}6$ (or 0·28), a 5 per cent shift in the position of no shear reduces the ratio $f_{1(2)}/f_1$ by 10 per cent on the smooth side and increases it by 5 per cent on the rough side after taking into account the changes in Reynolds number and ratio of rib height to equivalent diameter produced by the shift. Corresponding values for $f_2/f_1 = 1{\cdot}5$ are $6\frac{1}{2}$ per cent and $3\frac{1}{4}$ per cent respectively. These changes are just about the amounts needed to reduce Fig. 8 to a horizontal line through unity. The shift required must of course reduce to zero as f_2/f_1 tends to unity since in the limit the flow becomes symmetrical about the centre line of the channel.

A 5 per cent difference between the positions of no shear and maximum velocity for f_2/f_1 equal to 3·6 and 1·5 is more than can be attributed to experimental error in locating the position of maximum velocity since the scatter of the points about the curve in Fig. 9 is less than 5 per cent.

A comparison is made in Fig. 6(a) between the friction factors obtained from the annular and rectangular channels having roughnesses with the same ratio of pitch to height. The annular data [2] is based on the square-ribbed surface results for a pitch-to-height ratio, p/e, of 9·4 but with a small adjustment for the difference in p/e made by using the rectangular-ribbed surface results. The rectangular channel data is that appertaining to the identically roughened plates. A discrepancy between the two sets of data may be expected for a number of reasons—inadequacy of the equivalent diameter concept, choice of the wrong base for calculating wetted perimeter and flow area, a choice which has an effect when a change of curvature is involved, and the effect of the smooth channel on the roughened surface transformed friction factor in the annular passage. This last effect is likely to make the roughened surface friction factor appear small, as occurred in the rectangular passage tests. The reduction is obviously a function of perimeter ratio, since for a ratio of smooth to rough perimeter of zero the reduction is zero by definition, and for a ratio of unity the reduction is about 5 per cent. In the annular tests for which the perimeter ratio was 2·17 the reduction must be at least 5 per cent and hence the discrepancy between annular and rectangular passage results is greater than at first appears in Fig. 6(a). The effect of basing the wetted perimeter and flow area on a surface through the rib tips rather than through their roots is to slightly increase rather than decrease the discrepancy. Therefore unless there are some large systematic experimental errors it would appear that the failure of the equivalent diameter concept is the principal cause of the difference between the two sets of results.

CONCLUSION

The investigation has demonstrated that although the application of the transformation theory produces a variation of friction factor with Reynolds number similar to that in a fully roughened passage it does give rise to a discrepancy in the absolute value of the friction factor of up to 10 per cent low when the surface is tested in the presence of a smoother surface and of up to 10 per cent high when tested in the presence of a rougher surface. This conclusion applies to flat surfaces roughened by transverse isolated square ribs having a ratio of pitch to height of 10 and has been derived from an experiment with non-identically roughened plates where friction factors were in the ratio of 5 to 1 or less.

A comparison with results obtained from an annular channel indicates that the equivalent diameter concept gives poor correlation for roughened surfaces.

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APPENDIX 1

Force Balance in Rectangular Channel

Consider a small element of fluid length dx and of cross-section $(y_1 - y_m)\,dz$ in the region $du/dz = 0$ (Fig. A1). Also at y_m, $du/dy = 0$. At these surfaces since the velocity gradients are zero it is assumed that there is no net momentum transfer across them and the pressure forces acting on the fluid are balanced by the surface shear at the wall.

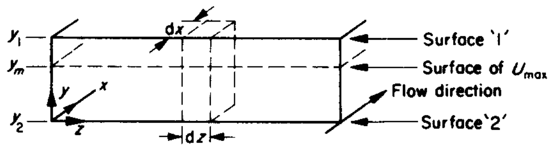

FIG. A1

Then

$$-dP\,(y_1 - y_m)\,dz = dz.\,dx.\,\tau_1$$

where τ_1 is the surface shear at y_1

$$\therefore\ \tau_1 = -\frac{dP}{dx}(y_1 - y_m).$$

A friction factor is defined by the expression $f = \tau/\frac{1}{2}\,\bar{\rho}\,\bar{u}^2$

$$\therefore f_1 = -(y_1 - y_m)\frac{dP}{dx}\Big/\tfrac{1}{2}\,\bar{\rho}_1\,\bar{u}_1^2$$

where $\bar{u}_1$ is the mean velocity in the element of of fluid.

For the rectangular channel under isothermal conditions

$$\frac{dP}{dx} = \frac{\Delta P}{\Delta x}\,(1 - \rho\bar{u}^2/\bar{P})$$

where $\Delta P/\Delta x$ is the measured static pressure gradient at the traverse point and the term in parentheses is a correction for acceleration; $\bar{P}$ is the mean pressure at the traverse point.

Similarly

$$\therefore f_2 = -(y_m - y_2)\frac{dP}{dx}\bigg/\tfrac{1}{2}\bar{\rho}_2\,\bar{u}_2^2.$$

A Reynolds number is defined for each friction factor as

$$Re_1 = \frac{\rho_1\,\bar{u}_1\,d_1}{\mu_1}$$

where $$d_1 = 4(y_1 - y_m)$$

$$Re_2 = \frac{\rho_2\,\bar{u}_2\,d_2}{\mu_2}$$

where $d_2 = 4(y_m - y_2)$.

APPENDIX 2

Discussion of the Significance Tests Applied in Order to Detect Departures from Transformation Theory

For the purposes of the significance tests the results have been divided into eight groups defined by f_2/f_1 greater than or less than unity for each of the four Reynolds numbers. The main reason for the division is that the five or six points in each group relate to different configurations and are therefore independent estimates of the effect of one surface on another. The results of the tests are listed in Table A1.

*Table A.*1

Reynolds number	$f_2/f_1 < 1$	$f_2/f_1 > 1$
2×10^4	$5\frac{1}{2}$%	$5\frac{1}{2}$%
5×10^4	14%	7%
1×10^5	7%	13%
$1{\cdot}5 \times 10^5$	1%	$7\frac{1}{2}$%

Significance levels indicating the breakdown of the transformation theory

Of the eight tests, six give a significance level better than 10 per cent, indicating that the theory does not hold. The departure from theory would be even more significant than indicated if, instead of testing the difference between the mean of each group and the expected value of unity against the scatter of the results about the mean, some account were taken, when calculating the standard deviation, of the fact that the curve is continuous throughout the range of f_2/f_1.

The significance test used in each case was a single-sided t-test. A single-sided test was adopted because it was anticipated that if the transformation theory were to break down it would break down in the way indicated by Fig. 8, i.e. an increase in friction factor when a rougher one is on the opposite wall and a decrease when a smoother one is there. The physical picture behind this hypothesis is that the higher turbulence level associated with the rougher surface would, if anything, lead to an increase in the turbulence level on the smoother side of the maximum velocity boundary and therefore to an increase in friction factor, its own value being reduced in the process.

Résumé—On a mesuré le coefficient de frottement dans une conduite rectangulaire formée essentiellement par deux plaques planes rendues rugueuses par des nervures isolées de section carrée. Lorsque les plaques n'ont pas la même rugosité, une théorie permet de séparer le coefficient de frottement de chaque surface et de l'appliquer à d'autres conduites ayant des nervures avec les mêmes rapports entre le pas et la hauteur et entre la hauteur et la diamètre équivalent. Les expériences indiquent que l'application de cette théorie entraîne des coefficients de frottement, qui, comparés avec ceux pour des plaques de rugosité identique pour lesquelles aucune transformation n'est nécessaire, varient de la même façon avec le nombre de Reynolds mais diffèrent en grandeur, de moins de 10 pour cent dans la gamme couverte. Les résultats présentent seulement un accord modéré avec des résultats analogues obtenus à partir d'une conduite annulaire.

Zusammenfassung—Messungen des Reibungsfaktors wurden an rechteckigen Kanälen durchgeführt, die im wesentlichen aus zwei ebenen Platten mit Rauhigkeiten aus einzelnen rechteckigen Querrippen bestanden. Wenn die Rauhigkeiten der Platten nicht identisch sind, erlaubt eine Theorie die getrennte Berechnung des Reibungsfaktors jeder Fläche und seine Anwendung auf andere Kanäle mit Rippen von gleichem Verhältnis von Teilung zu Höhe und Höhe zu äquivalentem Durchmesser. Die Untersuchungen zeigen, dass die Anwendung dieser Theorie Reibungsfaktoren ergibt, die sich verglichen mit jener für identische Rauhigkeit beider Platten, die keine Transformation erfordert, ähnlich der Reynoldszahl ändert aber in der Grösse unterscheidet. Diese Unterschiede betragen im untersuchten Bereich weniger als 10%. Die Ergebnisse zeigen nur mässige Übereinstimmung mit ähnlichen Ergebnissen für einen Kanal mit Ringquerschnitt.

Аннотация—Коэффициент трения измерялся в прямоугольном канале, сформированном двумя плоскими пластинами, шероховатость на которых создавалась поперечными изолированными квадратными ребрами. Для случая пластин с разной шероховатостью теория позволяет определить коэффициент трения для каждой поверхности и применить его к другим каналам, имеющим ребра с такими же отношениями p/e и e/d. Эксперименты показывают, что, применяя эту теорию, можно определить коэффициенты трения, которые зависят от Re также, как у непреобразованных пластин идентичной шероховатости и отличаются по величине не более, чем на 10% во всем исследуемом диапазоне. Результаты обнаруживают только относительное соответствие с данными, полученными для кольцевого канала.